

MATRICES (P1) (4 MARKS)

NOT A PART OF IGCSE SYLLABUS.

ORDER OF A MATRIX = ROWS $\times$ COLUMNS.

$$\begin{pmatrix} 3 & 2 & 7 \\ 4 & 1 & 6 \end{pmatrix}$$

2 $\times$ 3

$$\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$$

3 $\times$ 1

$$\begin{pmatrix} 2 & 3 \\ 5 & 6 \end{pmatrix}$$

2 $\times$ 2

$$\begin{pmatrix} 7 & 2 & 1 \end{pmatrix}$$

1 $\times$ 3

$$\begin{pmatrix} 9 & 1 \end{pmatrix}$$

1 $\times$ 2

$$\begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

2 $\times$ 1

ADD / SUB BOTH MATRICES MUST HAVE SAME ORDER

You can tell order is same by just looking.

$$1) \begin{pmatrix} 3 & 2 \\ 1 & 0 \\ 5 & 7 \end{pmatrix} + \begin{pmatrix} 4 & 0 \\ -1 & 2 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} \underline{7} & \underline{2} \\ \underline{0} & \underline{2} \\ \underline{8} & \underline{13} \end{pmatrix} \begin{array}{l} 1+(-1) \\ 1-1 \\ 0 \end{array}$$

$$2) \begin{pmatrix} 2 & -1 \\ 3 & -4 \end{pmatrix} - \begin{pmatrix} -2 & 4 \\ 3 & -5 \end{pmatrix} = \begin{pmatrix} \underline{4} & \underline{-5} \\ \underline{0} & \underline{1} \end{pmatrix}$$

$2 - (-2) = 2 + 2 = 4$
 $-1 - 4 = -5$
 $-4 - (-5) = -4 + 5 = 1$

MULTIPLY

(a) SCALAR MULTIPLICATION:

Multiply a matrix with a number.

$$(i) 3 \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ 15 \end{pmatrix}$$

Be Careful: FRACTIONS

$$\frac{3}{1} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = \frac{6}{5}$$

$$(ii) 2 \begin{pmatrix} 3 & -1 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & -2 \\ 0 & 8 \end{pmatrix}$$

MATRIX MULTIPLICATION

(This is working that we need in MATRIX TRANSFORMATION)

$$(i) \begin{pmatrix} \overset{U}{2} & \overset{L}{1} \\ \overset{D}{3} & \overset{M}{5} \end{pmatrix} \begin{pmatrix} \overset{L}{1} & \overset{M}{0} & \overset{R}{3} \\ \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow \end{pmatrix}$$

$2 \times 2 \quad \checkmark \quad 2 \times 3$

Ans Matrix = 2×3

$$\begin{pmatrix} \underline{3} & \underline{4} & \underline{12} \\ \underline{8} & \underline{20} & \underline{39} \end{pmatrix}$$

STEPS

1) Write the order of both matrices and check if they are allowed to multiply.

2) Make empty Ans Matrix

3) Mark Arrows

FIRST $\longrightarrow$

$$(T)(T) + (H)(H)$$

$$UL = (2)(1) + (1)(1) = 3$$

$$UM = (2)(0) + (1)(4) = 4$$

$$UR = (2)(3) + (1)(6) = 12$$

$$DL = (3)(1) + (5)(1) = 8$$

$$DM = (3)(0) + (5)(4) = 20$$

$$DR = (3)(3) + (5)(6) = 39$$

SECOND ↓

and name them.

4) Now MULTIPLY

TT + HH.

(ii)

$$\begin{pmatrix} 0 & 3 & 1 & 2 \\ 5 & 6 & 8 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$$

$$2 \times \begin{pmatrix} 3 & \checkmark & 3 \end{pmatrix} \times 1$$

Ans = 2×1

Answer Matrix $\begin{pmatrix} 13 \\ 31 \end{pmatrix}$

$$TT + MM + HH$$

$$UL = (3)(3) + (1)(0) + (2)(2) = 13$$

$$DL = (5)(3) + (6)(0) + (8)(2) = 31$$

DIVIDE DIVISION IS NOT POSSIBLE IN MATRICES.

So, STUBBORN MATHS PPL THOUGHT OF ANOTHER WAY.

eg: $AX = B$

$$X = \boxed{A^{-1}} B$$

Inverse of Matrix.

$$XA = B$$

$$X = BA^{-1}$$

Usually

$2 \times 3 = 3 \times 2$
are same

But in
matrices

$A \times B \neq B \times A$

INVERSE OF A 2×2 MATRIX

STEPS

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Main
Diagonal

i) Determinant:

Symbols: $|A|$ or $\det(A)$ or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$\det(A) = ad - bc$$

ii) Adjoint(A) = flip main diagonal
change other signs.

$$= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\text{iii) } A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} \text{Adjoint} \end{pmatrix}$$

eg $A = \begin{pmatrix} 1 & 3 \\ -2 & 4 \end{pmatrix}$ Find A^{-1} .

$$\begin{aligned} 1) \det(A) &= (1)(4) - (3)(-2) \\ &= 4 - (-6) \\ &= 10 \end{aligned}$$

$$2) \text{Adj}(A) = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$$

$$3) A^{-1} = \frac{1}{10} \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$$

So if it was

$$\frac{1}{10} \begin{pmatrix} 200 & -20 \\ -40 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & -2 \\ -4 & 1 \end{pmatrix}$$

This is where you must take $\frac{1}{10}$ inside bracket and simplify

IDENTITY MATRIX

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

1] If we multiply any matrix with Identity Matrix that matrix stays unchanged.

$$A \times I = A$$

$$\begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$$

2] If we multiply a matrix with its own inverse, the answer is Identity matrix.

$$A \times A^{-1} = I$$

EXAM QUESTION

(i) Simplify $\underbrace{AA^{-1}} \begin{pmatrix} 7 \\ 6 \end{pmatrix}$ (1 mark) (ii)

$$\begin{aligned} & \downarrow \\ & I \begin{pmatrix} 7 \\ 6 \end{pmatrix} \\ & = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} & 3 \times \underbrace{B \times B^{-1}} \\ & \downarrow \\ & 3 \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ & \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \end{aligned}$$